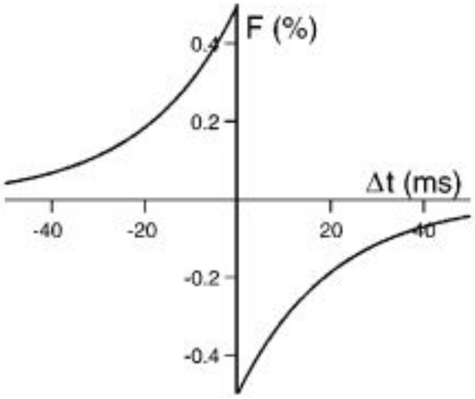
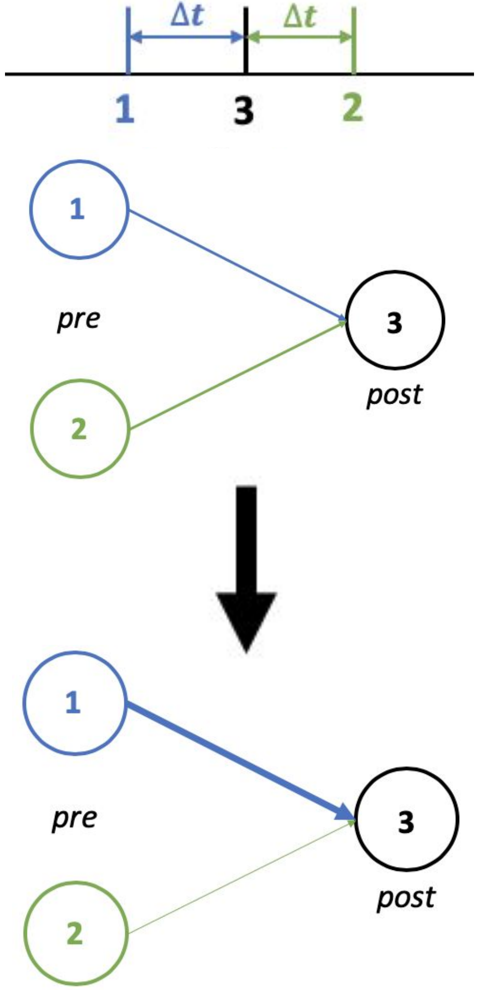
# Spike-Timing Dependent Plasticity

Nishant Mysore and Margot Wagner

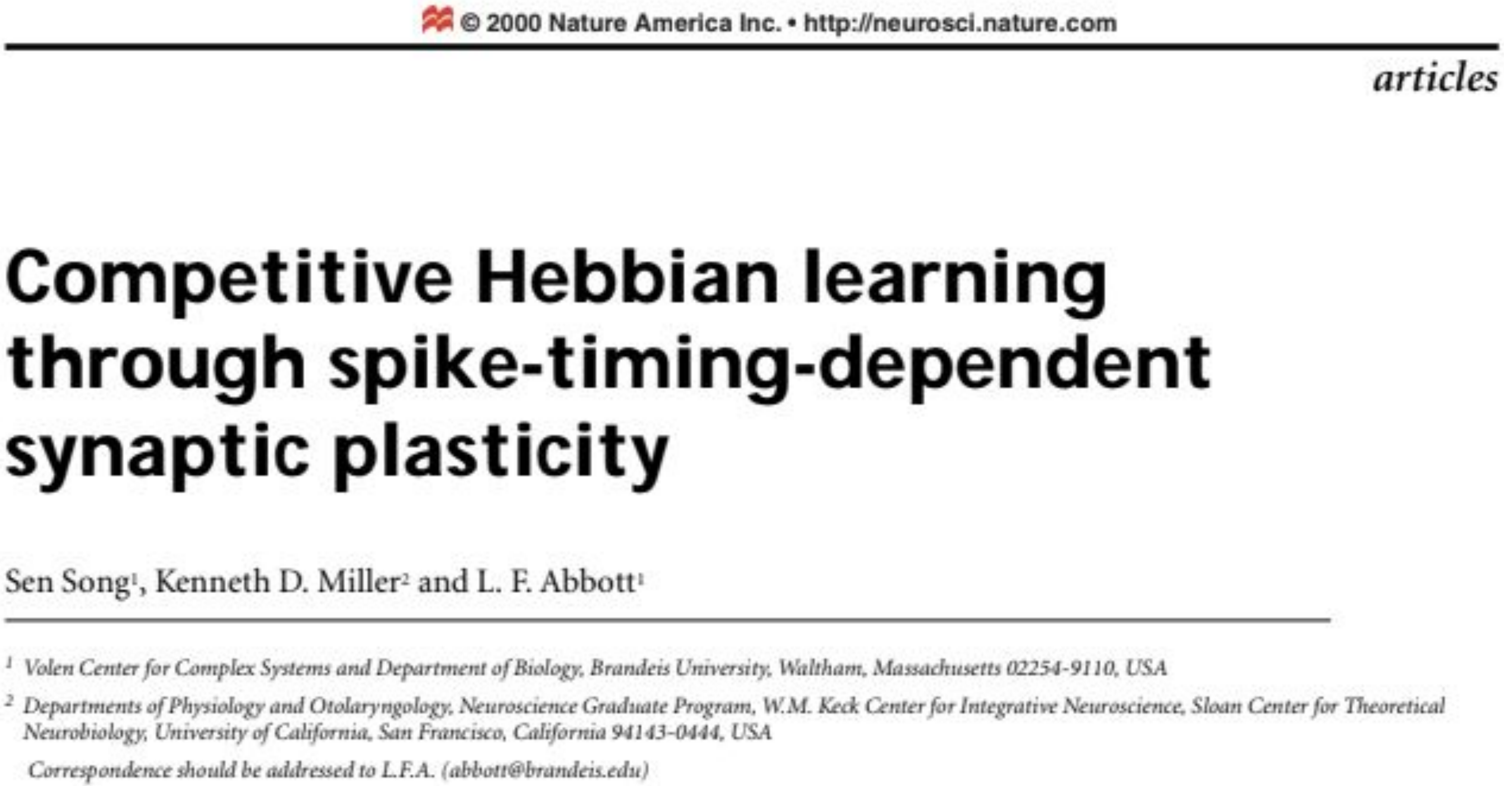
# Background

- Synaptic plasticity depending on relative timing of input and output action potentials
- Different types
- Classically strengthens for “pre before post” and weakens for “post before pre”
- Seen in hippocampus, neocortex, cerebellum, etc.





# Song *et al.* 2000



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## Competitive Hebbian learning through spike-timing-dependent synaptic plasticity

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# Methods

**LIF output neuron**

$$\tau_m \frac{dV_m}{dt} = V_{rest} - V_m + \overset{\text{excitatory input}}{\underline{g_{ex}(t)(E_{ex} - V_m)}} + \overset{\text{inhibitory input}}{\underline{g_{in}(t)(E_{in} - V_m)}}$$

**Action potential arrival**

$$g_{ex}(t) \rightarrow g_{ex}(t) + \overline{g}_a$$

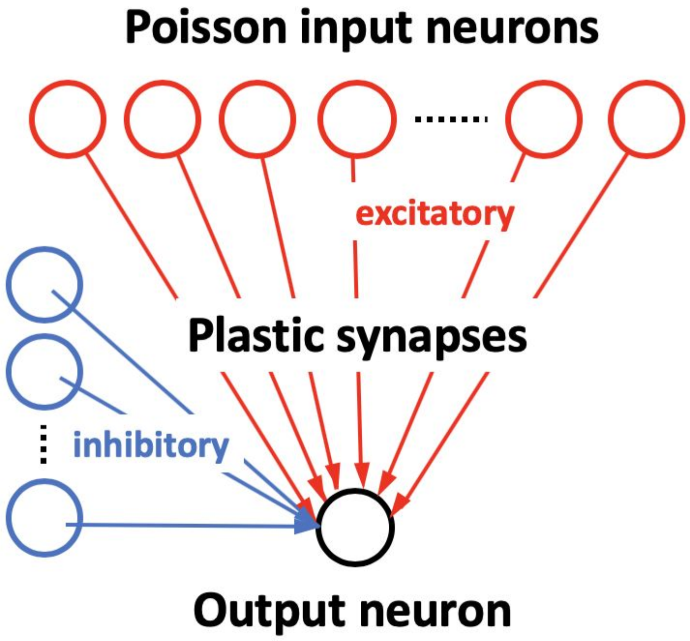
$$\downarrow$$

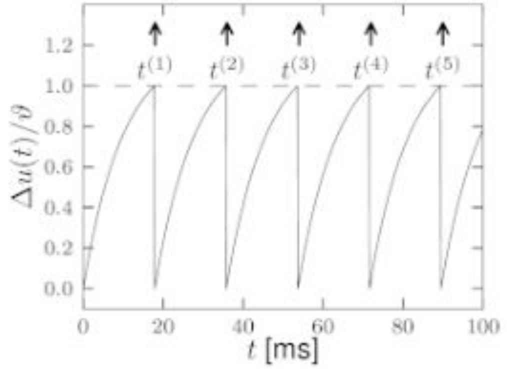
$$\tau_{ex} \frac{dg_{ex}}{dt} = -g_{ex}$$

$$g_{in}(t) \rightarrow g_{in}(t) + \overline{g}_{in}$$

$$\downarrow$$

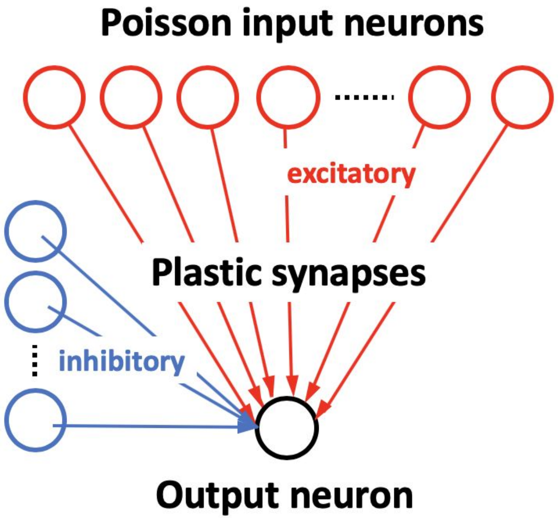
$$\tau_{in} \frac{dg_{in}}{dt} = -g_{in}$$





# Methods

$$F(\Delta t) = \begin{cases} A_{+} exp(\Delta t/\tau_{+}) & \text{if } \Delta t < 0 \\ A_{-} exp(-\Delta t/\tau_{-}) & \text{if } \Delta t \geq 0 \end{cases}$$



**Poisson input neurons**

**excitatory**

**Plastic synapses**

**inhibitory**

**Output neuron**

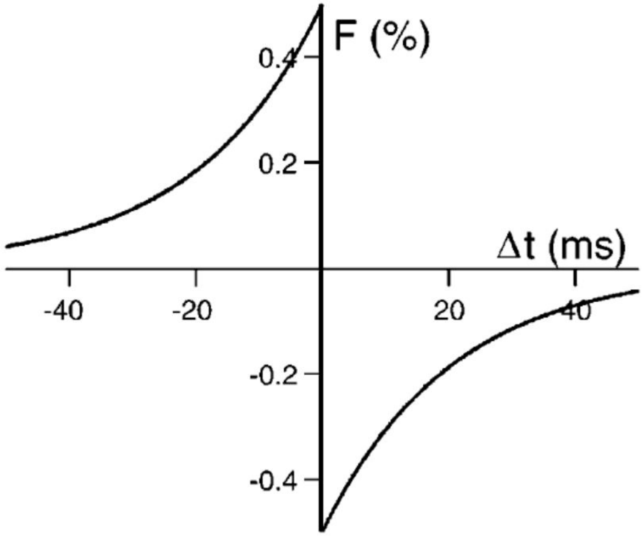
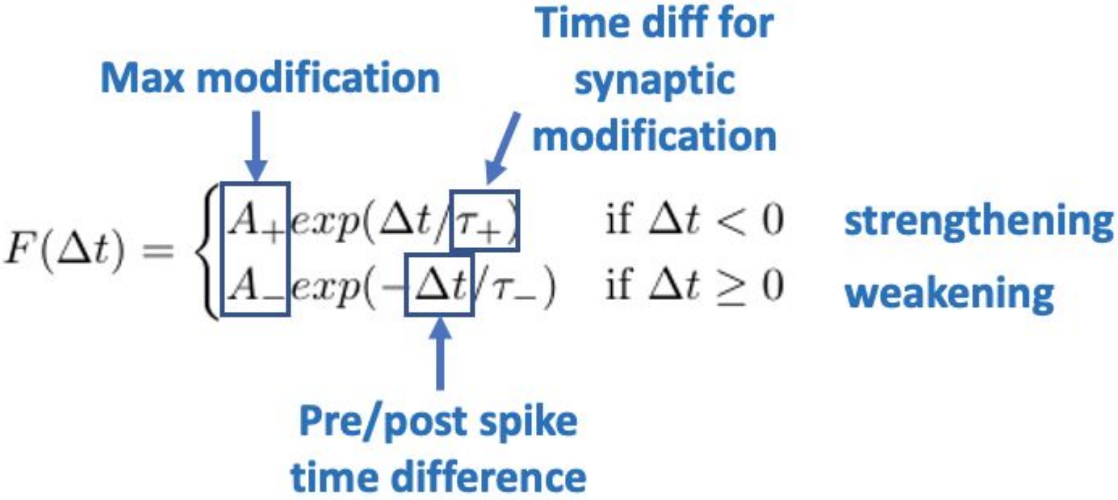


Figure 1: Synaptic modification function

# Methods



**Max modification**

**Time diff for synaptic modification**

$$F(\Delta t) = \begin{cases} \boxed{A_+} exp(\Delta t/\boxed{\tau_+}) & \text{if } \Delta t < 0 \\ \boxed{A_-} exp(-\boxed{\Delta t}/\tau_-) & \text{if } \Delta t \geq 0 \end{cases}$$

**strengthening**

**weakening**

**Pre/post spike time difference**

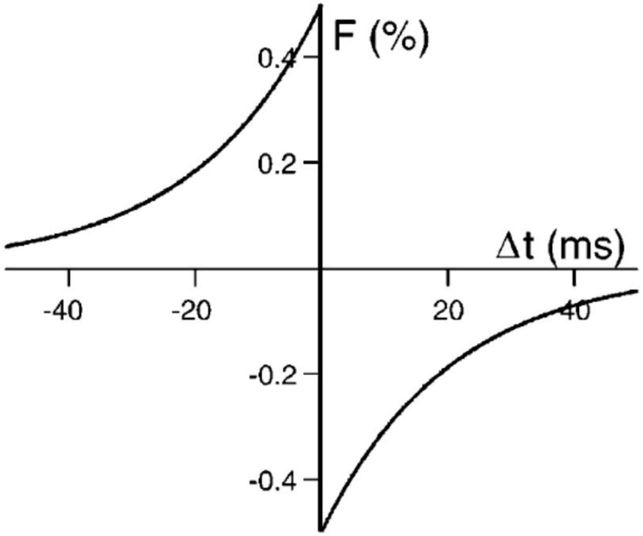


Figure 1: Synaptic modification function

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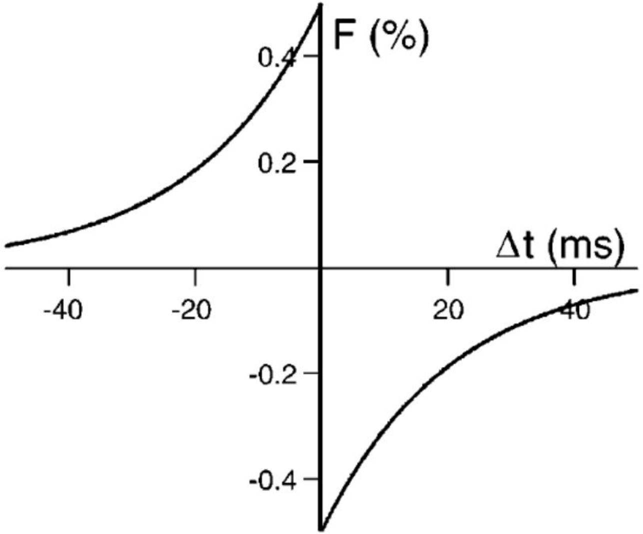


Figure 1: Synaptic modification function

- Change in **conductance** according to

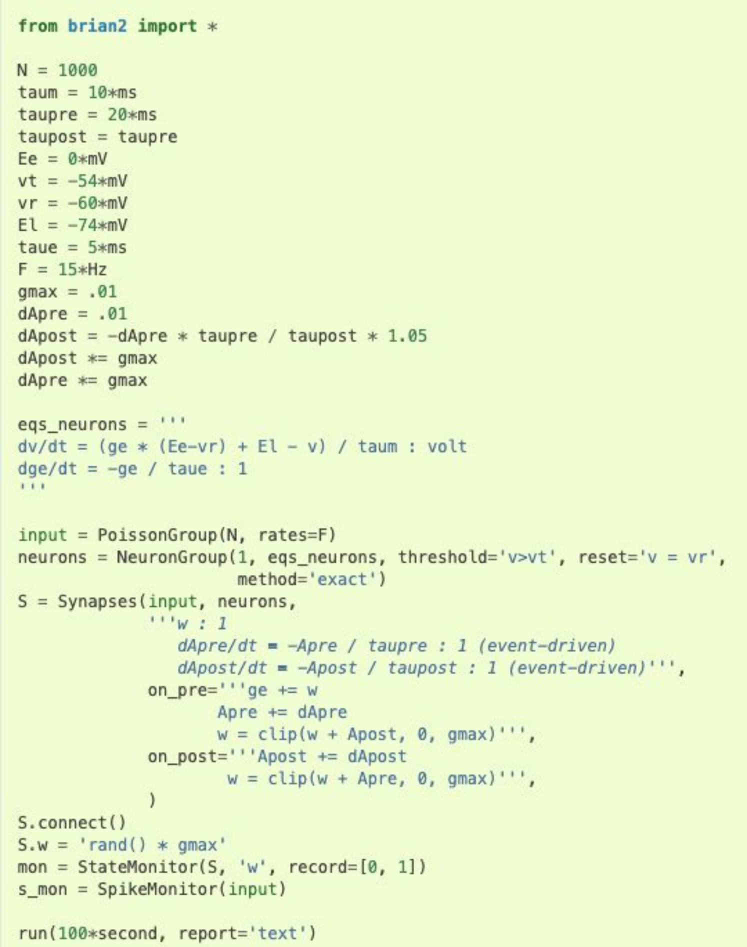
$$\overline{g}_a = \overline{g}_{max} F(\Delta t)$$

$$g_{ex}(t) \rightarrow g_{ex}(t) + \overline{g}_a$$

# Methods

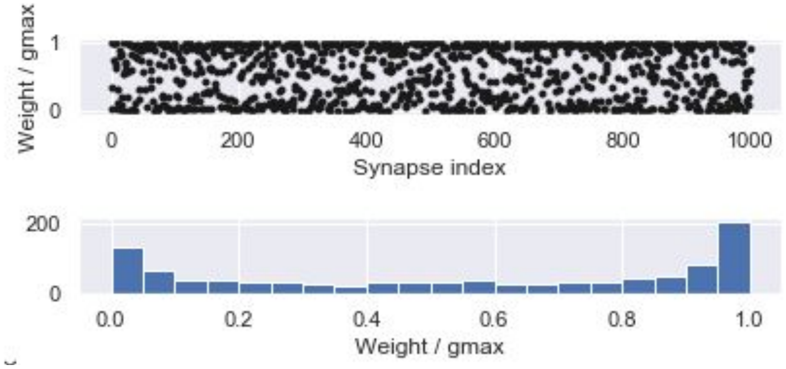
- Spiking neural network simulator

- Written in Python

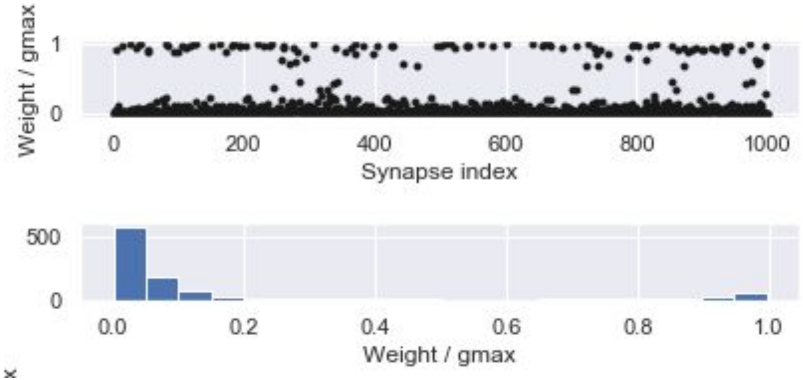


```
from brian2 import *

N = 1000
taum = 10*ms
taupre = 20*ms
taupost = taupre
Ee = 0*mV
vt = -54*mV
vr = -60*mV
El = -74*mV
taue = 5*ms
F = 15*Hz
gmax = .01
dApre = .01
dApost = -dApre * taupre / taupost * 1.05
dApost *= gmax
dApre *= gmax

eqs_neurons = '''
dv/dt = (ge * (Ee-vr) + El - v) / taum : volt
dge/dt = -ge / taue : 1
'''

input = PoissonGroup(N, rates=F)
neurons = NeuronGroup(1, eqs_neurons, threshold='v>vt', reset='v = vr',
                      method='exact')
S = Synapses(input, neurons,
             '''w : 1
                dApre/dt = -Apre / taupre : 1 (event-driven)
                dApost/dt = -Apost / taupost : 1 (event-driven)''',
             on_pre='''ge += w
                    Apre += dApre
                    w = clip(w + Apost, 0, gmax)''',
             on_post='''Apost += dApost
                     w = clip(w + Apre, 0, gmax)''',
             )
S.connect()
S.w = 'rand() * gmax'
mon = StateMonitor(S, 'w', record=[0, 1])
s_mon = SpikeMonitor(input)

run(100*second, report='text')
```

# Results



Excitatory Firing Rate = 10 Hz

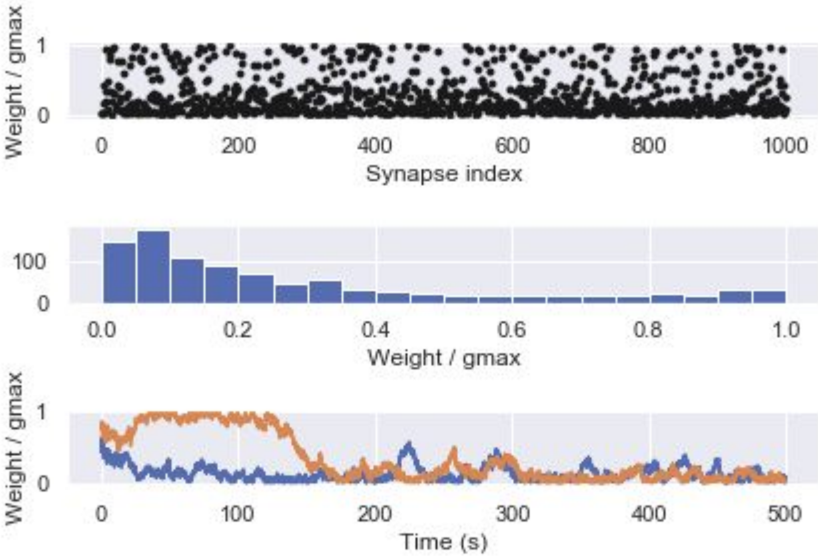


Excitatory Firing Rate = 40 Hz

- Simulation time 500 seconds
- Peak synaptic conductances have been pushed to extreme values
- Lower input frequencies push to the upper limit (bimodal) and higher input frequencies push to the lower limit.

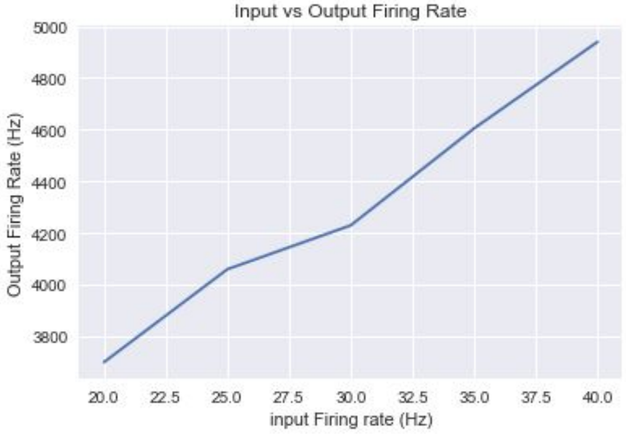
# Results

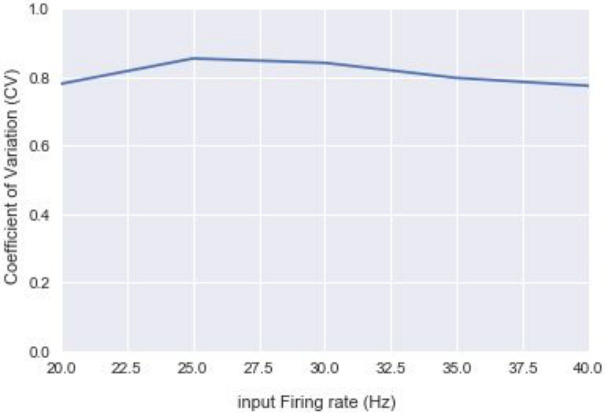
- Same simulation ran at 2.33 x $g_{max}$ values and 4 x the synaptic modification per spike pair ($A_+$ and A)



- The higher value of $g_{max}$ causes more synapses to lower conductance values, while the higher synaptic modification rate allows for conductance values throughout the entire range of possible conductance values.

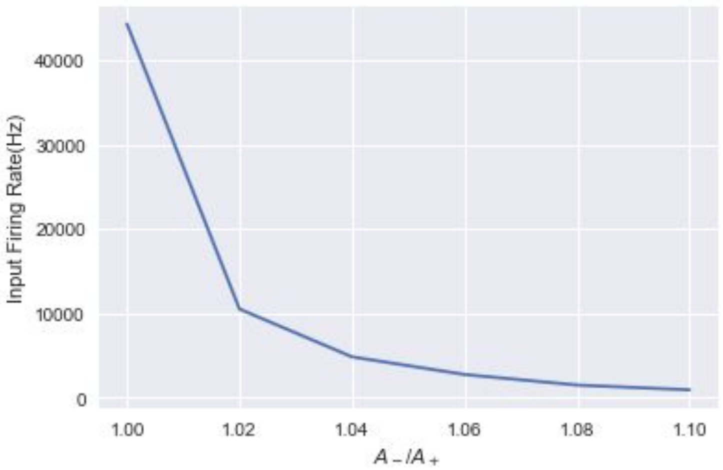
# Results

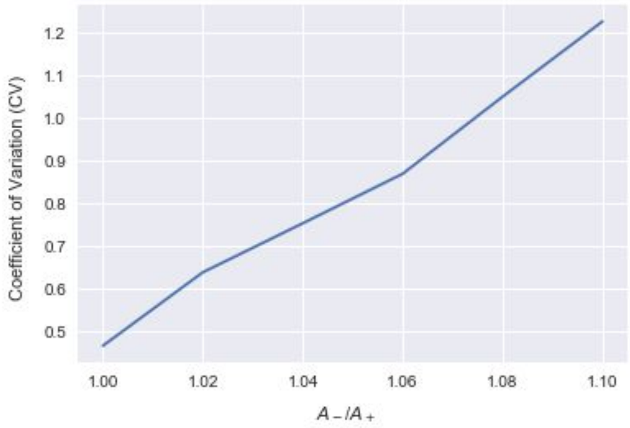




- Fairly linear increase between Excitatory input firing rate and output firing rate
- Coefficient of Variation is defined as std/mean of the Inter-Spike Interval
- STDP regulates the variability of postsynaptic response

# Results





- $A_-/A_+$ is the ratio of amplitudes of maximal synaptic strengthening and weakening
- If the ratio of $A_-$ to $A_+$ increases, the firing rate of the postsynaptic neuron decreases
- The CV starts to approach one, which indicates irregularity in the spike train

# Discussion

- Main problems with Hebbian modification
  - Synapses modified whenever correlated pre and postsynaptic activity occurs
  - Synapses don’t compete with each other
- STDP solves these problems:
  - Non-causal coincidences will weaken synapses if the integral of the synaptic modification function is negative
  - Competition is found through predicting timing of the postsynaptic action potential
- We have shown STDP leads to a balanced and irregular firing state
  - Correlation of Pre and Postsynaptic spike times
  - Information encoded specifically in the timing of the spikes
- STDP regularizes both the rate and CV of postsynaptic firing

# Discussion

- In our model of STDP, we’ve made several simplifications
  - Effects of Spike pairs sum linearly
  - Ignored delays between pairing of pre and postsynaptic spikes
  - Bounding conductance from 0 to $g_{max}$
- Limits of STDP
  - STDP cannot strengthen synapses in the absence of postsynaptic firing
  - If the excitatory synapses are too weak, STDP cannot rescue them.
  - Currently, two inputs that fire within 100 msec of each other won’t compete.

# References

- Song, S., Miller, K. & Abbott, L. Competitive Hebbian learning through spike-timing-dependent synaptic plasticity. *Nat Neurosci* 3, 919–926 (2000). https://doi.org/10.1038/78829
- Stimberg, M, Brette, R, Goodman, DFM. “Brian 2, an Intuitive and Efficient Neural Simulator.” eLife 8 (2019): e47314. doi: 10.7554/eLife.47314.